

Final Technical Report

for

**Hydrodynamic Analyses and Evaluation of  
New Fluid Film Bearing Concepts**

NASA Grant Number  
NCC3-436

Grant Duration  
November 15, 1995 to March 18, 1997

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December 1998

# Hydrodynamic Analyses and Evaluation of New Fluid Film Bearing Concepts

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## Introduction

Over the past several years, numerical and experimental investigations have been performed on a waved journal bearing. The research work was undertaken by Dr. Florin Dimofte, a Senior Research Associate in the Mechanical Engineering Department at the University of Toledo. Dr. Theo Keith, Distinguished University Professor in the Mechanical Engineering Department was the Technical Coordinator of the project.

The wave journal bearing is a bearing with a slight but precise variation in its circular profile such that a waved profile is circumscribed on the inner bearing diameter. The profile has a wave amplitude that is equal to a fraction of the bearing clearance.

Prior to this period of research on the wave bearing, computer codes were written and an experimental facility was established. During this period of research considerable effort was directed towards the study of the bearing's stability. The previously developed computer codes and the experimental facility were of critical importance in performing this stability research. A collection of papers and reports were written to describe the results of this work. The attached captures that effort and represents the research output during the grant period.

# Predicted and Experimentally Observed Fluid Film Instability of an Unloaded Gas Journal Bearing

by: Dr. Florin Dimofte

The University of Toledo, currently working at NASA Lewis  
Research Center in Cleveland, Ohio.

## ABSTRACT

Unloaded, plain, gas journal bearings are sensitive to a fluid film instability phenomena known as "Half Frequency Whirl Motion". The threshold at which this motion occurs was predicted using a numerical code based on a small perturbation technique. To substantiate the result of this numerical analysis, experiments were conducted using both a plain journal bearing and a bearing with a slight but precise variation in the circular profile of a journal bearing. The experimentally determined one-half frequency whirl motion thresholds agreed well with, and confirmed the theoretical predictions. Moreover, this movement is unstable for a plain journal bearing and contact between the shaft and the bearing wall was made shortly after the onset of the whirl motion in all cases for this bearing. Unlike the plain journal bearing, a bearing with a slight, but precise variation in the circular profile of a journal bearing shows a range of speeds under which the bearing can run free of half frequency whirl movement. Furthermore, when this movement does occur, the radius of the half frequency whirl motion increases up to a size at which equilibrium between the radial force generated by the whirl movement and the pressure force generated by the fluid film is established. This equilibrium radius is smaller than the bearing clearance and the bearing can run stably and safely, containing the half frequency movement.

## INTRODUCTION

The whirl motion of the shaft inside a gas journal bearing is a result of an unstable condition of the lubricant fluid film. The frequency of this motion is, in most of the cases, close to one-half of shaft frequency and is called "Half Frequency Whirl" (HFW). Unloaded plain journal bearings are very susceptible to the HFW. Plain journal bearings experiencing HFW usually develop an unstable motion that is an unsafe condition where the shaft comes in contact with the bearing often resulting in bearing failure. This phenomena was observed soon after gas bearing applications were developed starting in the middle 1950's. Due to its importance for bearing life the HFW was also well analyzed as a fluid film instability condition. Among others, Castelli and Elrod [1] and Constantinescu [2] contributed work that theoretically established when HFW occurs.

HFW can be avoided, at least for a specified range of the shaft running speeds, by changing the bearing profile using grooves, holes, lobes, etc. A new bearing profile which reduces the plane journal bearing sensitivity to HFW is a bearing with a slight, but precise variation in the circular profile of a journal bearing such that a waved profile circumscribed on the inner bearing diameter and having a wave amplitude equal to a fraction of the bearing clearance. A numerical code, based on a small perturbation

analysis, was developed to quantify the performance of the wave journal bearings and contrast it to plain journal bearings [3]. The numerical analysis revealed a significant difference in the hydrodynamic pressures generated in the lubricant by the wave and plain journal bearings which allows the wave bearing to have a higher static stiffness and better stability than the plain journal bearing.

## BEARING STABILITY

In a bearing stability calculation, as a first approach, the rotor can be considered rigid and symmetrical, and supported by two identical bearings. Therefore, each bearing carries a mass  $M$  that is one-half of the rotor mass. The motion equation of the center of the shaft inside the bearing can be written as:

$$\begin{bmatrix} (Z_{xx} - M v^2) & Z_{xy} \\ Z_{yx} & (Z_{yy} - M v^2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

where the bearing is represented by its four impedance coefficients,  $Z_{jk}$  ( $j = x, y$ ;  $k = x, y$ ) which can be calculated by using a small perturbation technique [4], and  $v$  is the whirl frequency.

The threshold of instability occurs when the determinant of the matrix is zero and the corresponding mass,  $M_c$ , is the mass required to develop the HFW movement of the shaft inside the bearing. The bearing can run free of half frequency whirl movement when its actual allocated mass is less than the critical mass.

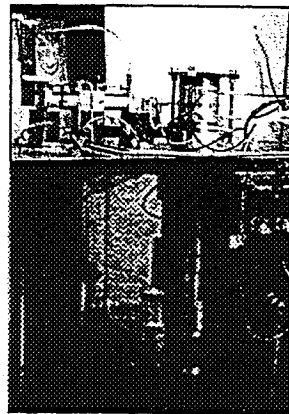


FIG. 1 TEST RIG

## TEST RIG

To substantiate the results of the numerical analysis, a bench test rig was assembled to measure the steady-state performance of journal air bearings and to provide information about how the bearing behaves when HFW movement

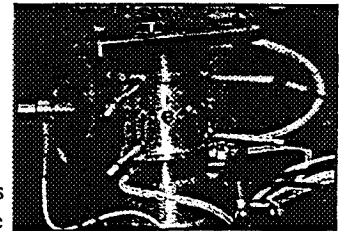


FIG.2 HFW TEST BEARING

occurs. The bearing tester uses a commercial spindle capable of 30,000 RPM with a run-out of less than 1 micron. The rig was set up with the shaft oriented vertically such that the test bearing could easily run in an unloaded condition (Fig. 1). Air at surrounding atmospheric conditions, was used as the lubricant. The test bearing is supported by double thrust

started to turn (e.g. 355 RPM). The motion was unstable because its orbit increased very rapidly and the shaft rubbed the bearing wall as can be seen in the bottom screen of Fig. 5. The upper screen of Fig. 5 shows the tracks of two displacement sensors, one in the top and the other in the bottom, fixed in the same vertical plane. Both of these tracks followed the same path showing a translatory movement of the bearing which is close to a harmonic motion. The frequency of the bearing motion was 2.96 Hz or half of the shaft frequency.

Then, both a three- and a two-wave journal bearing were tested. The three-wave bearing had a radial clearance of .037 mm and a corresponding ratio of the wave amplitude ratio to the radial clearance of 0.343. This bearing ran free of SSFW up to a threshold speed between 740 and 960 RPM. When the radial clearance was reduced to 0.015 mm the three-wave bearing ran free of SSFW up to a speed of 11900 RPM. The wave amplitude ratio was, in this case, only 0.168. These experimentally observed thresholds of SSFW were in very good agreement with the theoretical prediction [6]. Moreover, as speed increases, the whirl orbit is stable keeping a safe range well within the bearing clearance (see bottom screen in Fig. 6). The three-wave bearing profile limits the maximum amplitude of the shaft center (upper screen of Fig. 6) that modifies the shape of the orbit from circular to a three side orbit (bottom screen of Fig. 6).

The test of a two-wave bearing with 0.038 mm radial clearance and a wave amplitude ratio of 0.442 shows that the bearing operates very stable at intermediary speeds such as 10000 RPM (Fig. 7) while experiencing SSFW at both low and high speeds such as 600 and 24000 RPM respectively (Fig. 8 and Fig. 9). When the SSFW occurs the bearing runs stable in a manner similar to the three wave bearing. However the dominant sub-synchronous whirl frequency, in this case, is close to a quarter of the synchronous shaft speed. In addition the half frequency and moreover the synchronous frequency are also present as can be better seen in the upper part of Fig. 9. When the wave amplitude ratio was decreased to 0.214, the whirl frequency increases close to half of the synchronous frequency like in both plain and three wave journal bearings.

## CONCLUDING REMARKS

1. The observed thresholds at which the SSFW occurs correspond with the predicted thresholds.

2. An unloaded journal bearing with an altered circular profile, such as a wave bearing, allows operation over a range of speeds under which the bearing can run free of SSFW. When this movement occurs the radius of the SSFW increases up to a size where the equilibrium between the radial force generated by the whirl movement and the pressure force in the film is established. The equilibrium radius is smaller than the bearing clearance and the bearing can run stably and safely.

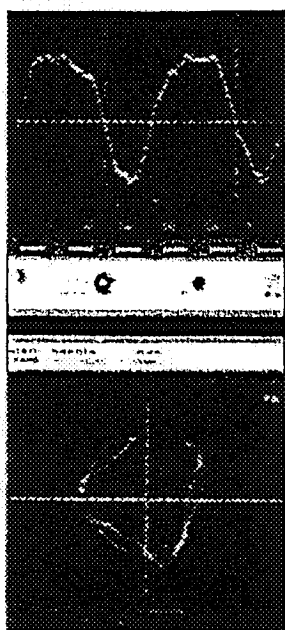


FIG. 6 SSFW OF A THREE-WAVE BEARING AT 1391 RPM, WHIRL FREQUENCY = 10.21 Hz

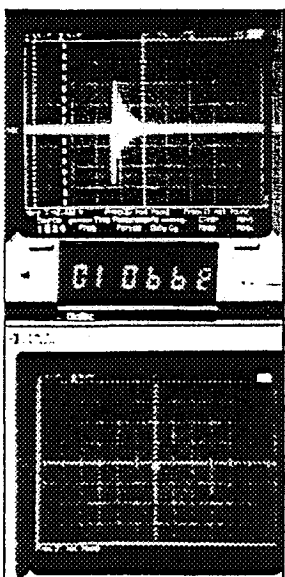


FIG. 7 SYNCHRONOUS WHIRL OF A TWO-WAVE BEARING AT 10668 RPM AFTER AN IMPACT.

3. At large clearances and wave amplitudes a two-wave bearing, unlike other bearings, can exhibit a sub-synchronous whirl movement at both low and high speeds, but can run extremely stable and without whirl at intermediate speeds. Moreover, in these cases, the whirl frequencies are close to a quarter of the synchronous speed. The three-wave bearing can exhibit the sub-synchronous whirl motion only after a specific threshold when the speed increases and the whirl frequencies are close to half of the synchronous speed. At smaller clearances both two- and three-wave bearings exhibit half synchronous whirl frequency and increased onset whirl thresholds.

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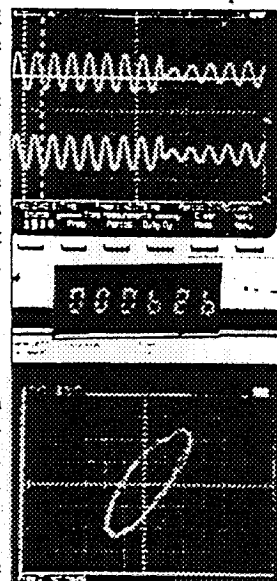


FIG. 8 SSFW OF A TWO-WAVE BEARING AT 626 RPM, WHIRL FREQUENCY = 2.31 (2.77) Hz

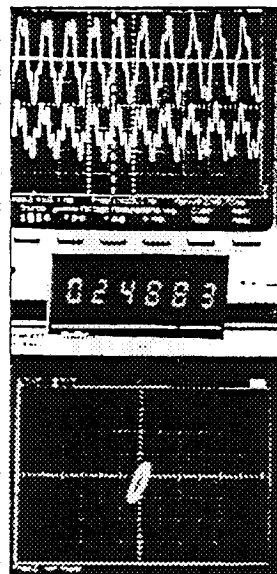


FIG. 9 SSFW OF A TWO-WAVE BEARING AT 24883 RPM, WHIRL FREQUENCY = 103.1, 206.2, AND 412.4 Hz

# Fractional Whirl Motion in Wave Journal Bearings

by: Dr. Florin Dimofte  
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Research Center in Cleveland, Ohio, and

Robert C. Hendricks,  
NASA Lewis Research Center, Cleveland, Ohio.

## ABSTRACT

Unloaded gas, plain journal bearings experience sub-synchronous whirl motion due to fluid film instabilities and wall contact usually occurs immediately after the onset of the whirl motion. An alternative is the wave journal bearing which significantly improves bearing stability. The predicted threshold where the sub-synchronous whirl motion starts was well confirmed by the experimental observation. In addition, both a two-wave and a three-wave journal bearing can operate free of sub-synchronous whirl motion over a large range in speeds. When the sub-synchronous whirl motion occurs, both the two-wave and three-wave bearing can run in a whirl orbit well within the bearing clearance. At large clearances and wave amplitudes a two-wave bearing, unlike other bearings, can exhibit a sub-synchronous whirl movement at both low and high speeds, but can run extremely stable and without whirl at intermediate speeds. Moreover, in these cases, the whirl frequencies are close to a quarter of the synchronous speed. The three-wave bearing can exhibit sub-synchronous whirl motion only after a specific threshold when the speed increases and the whirl frequencies are close to half of the synchronous speed.

## INTRODUCTION

The whirl motion of the shaft inside a gas journal bearing is a result of an unstable condition of the lubricant fluid film. The frequency of this motion is, in most cases, equal or less than one-half of shaft frequency and is called "Sub-Synchronous Frequency Whirl" (SSFW). Unloaded plain journal bearings are very susceptible to the SSFW. Plain journal bearings experiencing SSFW usually develop an unstable motion and wall contact occurs immediately after the onset of the whirl motion resulting in bearing failure. This phenomena was observed soon after gas bearing applications were developed starting in the middle 1950's. Due to its importance for bearing life the SSFW was also well analyzed as a fluid film instability condition. Among others, Castelli and Elrod [1] and Constantinescu [2] contributed work that theoretically established when SSFW occurs.

Unlike the plain journal bearing (Fig.1), a wave journal bearing (Fig.2 shows a three-wave bearing) reduces the journal bearing sensitivity to SSFW. A wave journal bearing [3] is a bearing with a slight, but precise variation in the circular profile of a journal bearing such that a waved profile is circumscribed on the inner bearing diameter and having a wave amplitude equal to a fraction of the bearing clearance. Fig. 2 shows a three-wave bearing. The clearance and the wave and the wave's amplitude are greatly exaggerated in Fig.1 and 2 so that the concept may be visualized.

## BEARING STABILITY

In a bearing stability calculation, as a first approach, the rotor can be

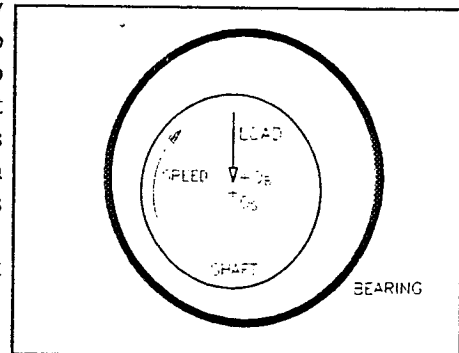


FIG. 1 PLAIN JOURNAL BEARING

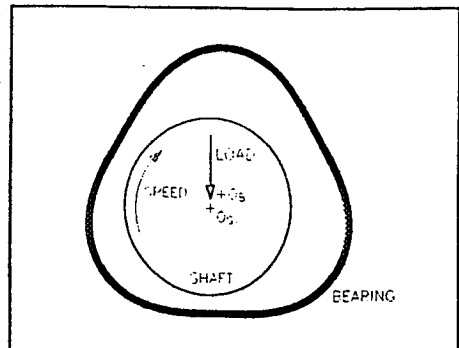


FIG. 2 THREE-WAVE JOURNAL BEARING

considered rigid and symmetrical, and supported by two identical bearings. Therefore, each bearing carries a mass  $M$  that is one-half of the rotor mass. The motion equation of the center of the shaft inside the bearing can be written as:

$$\begin{bmatrix} (Z_{xx} - M \nu^2) & Z_{xy} \\ Z_{yx} & (Z_{yy} - M \nu^2) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

where the bearing is represented by its four impedance coefficients,  $Z_{jk}$  ( $j = x, y; k = x, y$ ) which can be calculated by using a small perturbation technique [4], and  $\nu$  is the whirl frequency.

The threshold of instability occurs when the determinant of the matrix is zero and the corresponding mass,  $M_c$ , is the mass required to develop the SSFW movement of the shaft inside the bearing. The bearing can run free of half frequency whirl movement when its actual allocated mass is less than the critical mass ( $M_c$ ).

## TEST RIG

A bench test rig was assembled to provide information about how a journal bearing behaves when SSFW movement occurs [5]. The bearing tester uses a commercial spindle capable of 30,000 RPM with a run-out of less than 1 micron. The rig was set up with the shaft oriented vertically such that the test bearing could easily run in an unloaded condition (Fig. 3). Air at surrounding atmospheric conditions, was used as the lubricant. The test bearing is supported by double thrust levitation air bearing plates that allow the bearing to move very freely in the radial direction (Fig. 4). Two pairs of displacement sensors allow measurement of bearing orbits with respect to the shaft at two locations, just above and just below the test bearing. This configuration allows the bearing threshold susceptibility to SSFW to be precisely observed and provides information

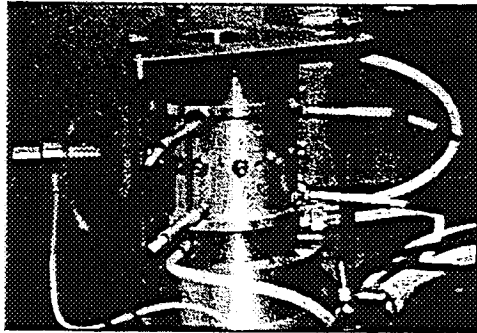


FIG. 4 BEARING FOR SSFW TEST

how the bearing behaves under SSFW.

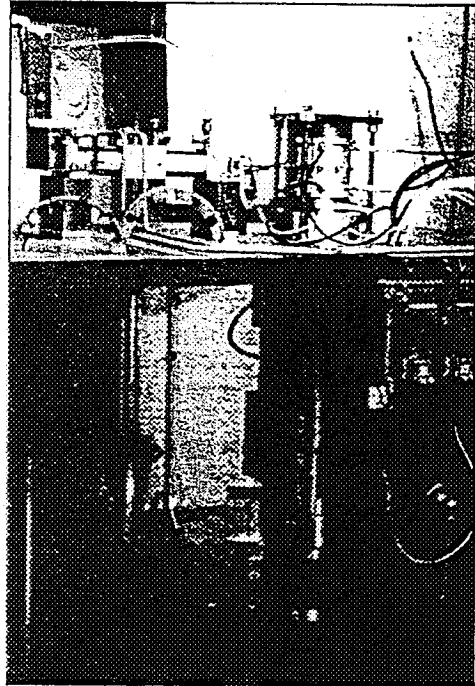


FIG. 3 TEST RIG

## RESULTS AND DISCUSSION

A plain journal bearing with 50 mm diameter, 58 mm length, and 0.038 mm radial clearance was tested [6]. The maximum out of round was 0.0012 mm which results in the ratio of the distortion amplitude to the radial clearance of 0.034. The test results confirm that the plain journal bearing is very susceptible to SSFW. The whirl movement appeared soon after the shaft started to turn (e.g. 355 RPM). The motion was unstable because its orbit increased very rapidly and the shaft rubbed the bearing wall as can be seen in the bottom screen of Fig. 5. The upper screen of Fig. 4 shows the tracks of two displacement sensors, one in the top and the other in the bottom, fixed in the same vertical plane. Both of these tracks followed the same path showing a translatory movement of the bearing which is close to a harmonic motion. The frequency of the bearing motion was 2.96 Hz or half of the shaft frequency.

Then, both a three- and a two-wave journal bearing were tested. The three-wave bearing had a radial clearance of .037 mm and a corresponding ratio of the wave amplitude ratio to the radial clearance of 0.343. This bearing ran free of SSFW up to a threshold speed between 740 and 960 RPM. One of the observed threshold can be seen in

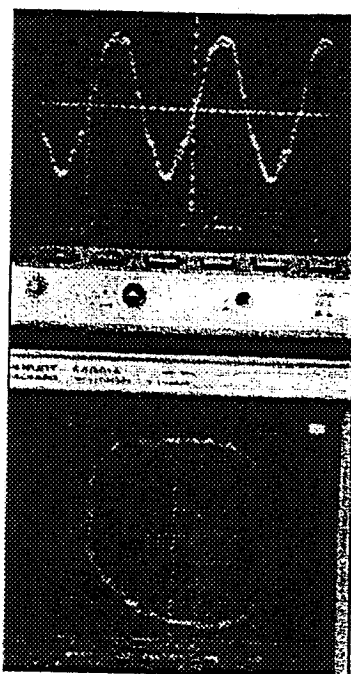


FIG. 5 SSFW OF A PLAIN BEARING AT 335 RPM, WHIRL FREQUENCY = 2.9 Hz

Fig. 6. When the radial clearance was reduced to 0.015 mm the three-wave bearing ran free of SSFW up to a speed of 11900 RPM. The wave amplitude ratio was, in this case, only 0.168. These experimentally observed thresholds of SSFW were in very good agreement with the theoretically prediction as can be seen in Fig. 7 [6]. Moreover, as speed increases, the whirl orbit is stable keeping a safe range well within the bearing clearance (see bottom screen in Fig. 8). The three-wave bearing profile limits the maximum amplitude of the shaft center (upper screen of Fig. 8) that modifies the shape of the orbit from circular to a three side orbit (bottom screen of Fig. 8).

The test of a two-wave bearing with 0.038 mm radial clearance and a wave amplitude ratio of 0.442 shows that the bearing operates very stable at intermediary speeds such as 10000 RPM (Fig. 9) while experiencing SSFW at both low and high speeds such as 600 and 24000 RPM respectively (Fig. 10 and Fig. 11). When the SSFW occurs the bearing runs stable in a manner similar to the three wave bearing. However the dominant sub-synchronous whirl frequency, in this case, is close to a quarter of the synchronous shaft speed. In addition the half frequency and moreover the synchronous frequency are also present as can be better seen in the upper part of Fig. 11. When the wave amplitude ratio was decreased to 0.214, the whirl frequency increases close to half of the synchronous frequency like in both plain and three wave journal bearings.

#### CONCLUDING REMARKS

1. The observed thresholds at which the SSFW occurs correspond with the predicted thresholds.
2. An unloaded journal bearing with an altered circular profile, such as a wave bearing, allows operation over a range of speeds under which the bearing can run free of SSFW. When this movement occurs the radius of the SSFW increases up to a size where the equilibrium between the radial force generated by the whirl movement and the pressure force in the film is established. The equilibrium radius is smaller than the bearing clearance and the bearing can run stably and safely.
3. At large clearances and wave amplitudes a two-wave bearing, unlike other bearings, can exhibit a sub-synchronous whirl movement at both low and high speeds, but can run extremely stable and without whirl at intermediate speeds. Moreover, in these cases, the whirl frequencies are close to a quarter of the synchronous speed. The three-wave bearing can exhibit the sub-synchronous whirl motion only after a specific threshold when the speed increases and the whirl frequencies are close to half of the synchronous speed.

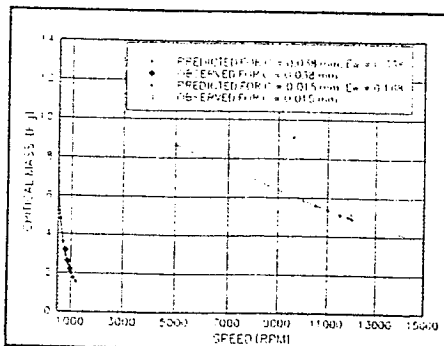


FIG. 7 PREDICTED AND OBSERVED THRESHOLD OF SSFW FOR THREE-WAVE BEARINGS.

4. At smaller clearances both two- and three-wave bearings exhibit half synchronous whirl frequency and increased onset whirl thresholds.

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1. Castelli, V. and Elrod, H.G., "Solution of the Stability Problem for 360 Deg Self-Acting, Gas-Lubricated Bearings," Journal of Basic

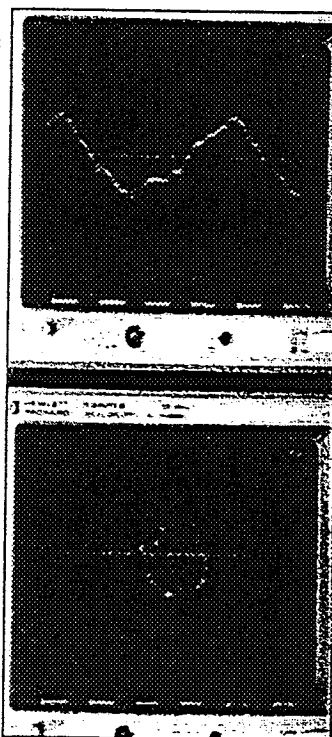


FIG. 6 THRESHOLD OF SSFW OF A THREE-WAVE BEARING AT 921 RPM, WHIRL FREQUENCY = 7.58 Hz.

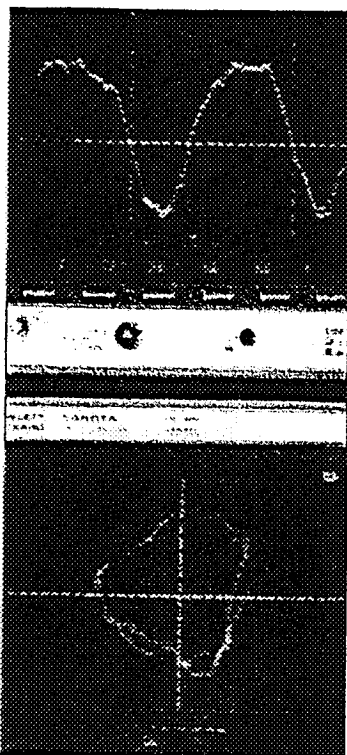


FIG. 8 SSFW OF A THREE-WAVE BEARING AT 1391 RPM, WHIRL FREQUENCY = 10.21 Hz

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5. Dimofte, F., Addy, H., E., Jr., and Walker, J., F., "Preliminary Experimental Results of a Three Wave Journal Air Bearing", Proceeding of Advanced Earth-to-Orbit Propulsion Technology Conference held at NASA Marshall Space Flight Center, Huntsville, AL, May 17-19, 1994, NASA CP 3282, Vol II, pp.375-384, 1994.

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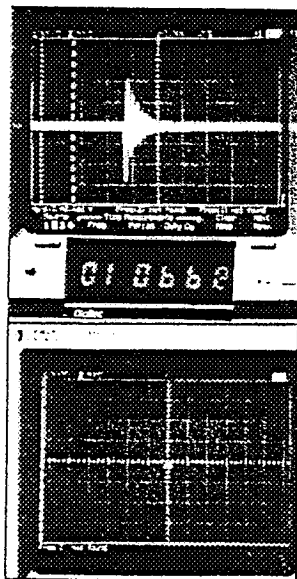


FIG. 9 SYNCHRONOUS WHIRL OF A TWO-WAVE BEARING AT 10668 RPM AFTER AN IMPACT.

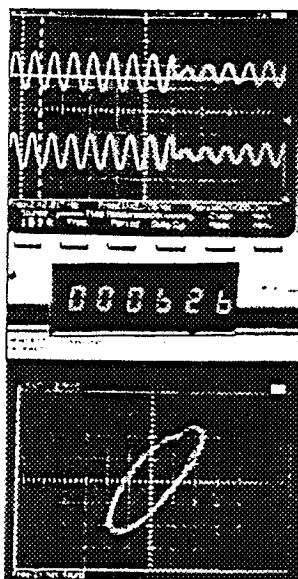


FIG. 10 SSFW OF A TWO-WAVE BEARING AT 626 RPM, WHIRL FREQUENCY = 2.81 (2.77) Hz

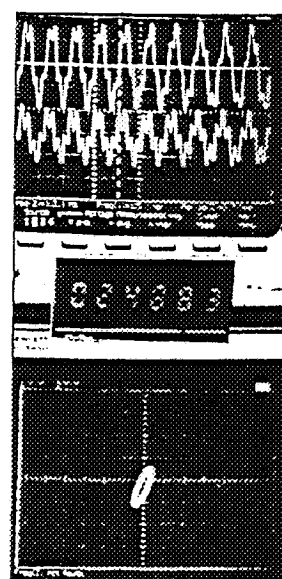


FIG. 11 SSFW OF A TWO-WAVE BEARING AT 24883 RPM, WHIRL FREQUENCY = 103.1, 206.2, AND 412.4 Hz



# Two- and Three- Wave Journal Bearing Fractional Whirl Motion

by: Dr. Florin Dimofte

The University of Toledo, currently working at NASA Lewis  
Research Center in Cleveland, Ohio, and  
Robert C. Hendricks,  
NASA Lewis Research Center, Cleveland, Ohio.

## ABSTRACT

Unloaded gas, plain journal bearings experience sub-synchronous whirl motion due to fluid film instabilities and wall contact usually occurs immediately after the onset of the whirl motion. An alternative is the wave journal bearing which significantly improves bearing stability. Both a two-wave and a three-wave journal bearing can operate free of sub-synchronous whirl motion over a large range in speeds. When the sub-synchronous whirl motion occurs, both the two-wave and three-wave bearing can run in a whirl orbit well within the bearing clearance. At large clearances and wave amplitudes a two-wave bearing, unlike other bearings, can exhibit a sub-synchronous whirl movement at both low and high speeds, but can run extremely stable and without whirl at intermediate speeds. Moreover, in these cases, the whirl frequencies are close to a quarter of the synchronous speed. The three-wave bearing can exhibit sub-synchronous whirl motion only after a specific threshold when the speed increases and the whirl frequencies are close to half of the synchronous speed.

## INTRODUCTION

The whirl motion of the shaft inside a gas journal bearing is a result of an unstable condition of the lubricant fluid film. The frequency of this motion is, in most cases, equal or less than one-half of shaft frequency and is called "Sub-Synchronous Frequency Whirl" (SSFW). Unloaded plain journal bearings are very susceptible to the SSFW. Plain journal bearings experiencing SSFW usually develop an unstable motion and wall contact occurs immediately after the onset of the whirl motion resulting in bearing failure. This phenomena was observed soon after gas bearing applications were developed starting in the middle 1950's. Due to its importance for bearing life the SSFW was also well analyzed as a fluid film instability condition. Among others, Castelli and Elrod [1] and Constantinescu [2] contributed work that theoretically established when SSFW occurs.

Unlike the plain journal bearing (Fig.1), a wave journal bearing (Fig.2 shows a three-wave bearing) reduces the journal bearing sensitivity to SSFW. A wave journal bearing [3] is a bearing with a slight, but precise variation in the circular profile of a journal bearing such that a waved profile is circumscribed on the inner bearing diameter and having a wave amplitude equal to a fraction of the bearing clearance. Fig. 2 shows a three-wave bearing. The clearance and the wave and the wave's amplitude are greatly exaggerated in Fig.1 and 2 so that the concept may be visualized.

## BEARING STABILITY

In a bearing stability calculation, as a first approach, the rotor can be considered rigid and symmetrical, and supported by two identical bearings.

Therefore, each bearing carries a mass  $M$  that is one-half of the rotor mass. The motion equation of the center of the shaft inside the bearing can be written as:

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where the bearing is represented by its four impedance coefficients,  $Z_{jk}$  ( $j = x, y; k = x, y$ ) which can be calculated by using a small perturbation technique [4], and  $\omega$  is the whirl frequency.

The threshold of instability occurs when the determinant of the matrix is zero and the corresponding mass,  $M_c$ , is the mass required to develop the SSFW movement of the shaft inside the bearing. The bearing can run free of half frequency whirl movement when its actual allocated mass is less than the critical mass ( $M_c$ ).

## TEST RIG

A bench test rig was assembled to provide information about how a journal bearing behaves when SSFW movement occurs [5]. The bearing tester uses a commercial spindle capable of 30,000 RPM with a run-out of less than 1 micron. The rig was set up with the shaft oriented vertically such that the test bearing could easily run in an unloaded condition (Fig. 2). Air at surrounding atmospheric conditions, was used as the lubricant. The test bearing is supported by double thrust levitation air bearing plates that allow the bearing to move very freely in the radial direction (Fig. 3). Two pairs of displacement sensors allow measurement of bearing orbits with respect to the shaft at two locations, just above and just below the test bearing. This configuration allows the bearing threshold susceptibility to SSFW to be precisely observed and provides information how the bearing behaves under SSFW.

## RESULTS AND DISCUSSION

A plain journal bearing with 50 mm diameter, 58 mm length, and 0.038 mm radial clearance was tested [6]. The maximum out of round was 0.0012 mm which results in the ratio of the distortion amplitude to the radial clearance of 0.034. The test results confirm that the plain journal bearing is very susceptible to SSFW. The whirl movement appeared soon after the shaft

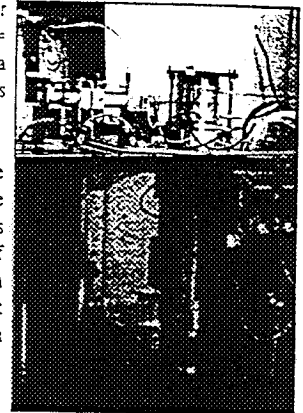


FIG. 3 TEST RIG

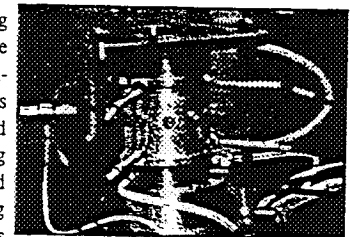


FIG. 4 BEARING FOR SSFW TEST

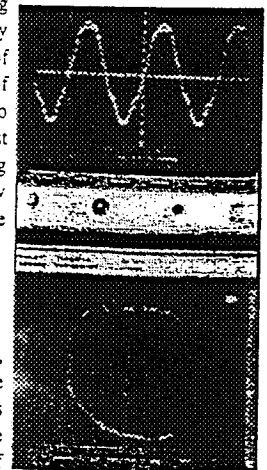


FIG. 5 SSFW OF A PLAIN BEARING AT 335 RPM, WHIRL FREQUENCY = 2.9 Hz

levitation air bearing plates that allow the bearing to move freely in the radial direction (Fig. 2). Two pairs of displacement sensors allow measurement of bearing orbits with respect to the shaft at two locations, just above and just below the test bearing. This configuration allows the bearing stability to be observed during the test and to precisely locate the threshold of HFW.

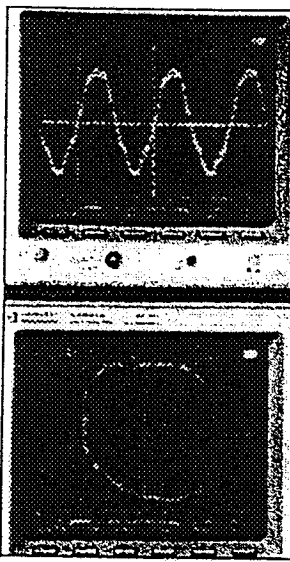


FIG. 3 HFW OF A PLAIN JOURNAL BEARING AT SPEED = 355 RPM

The upper screen of Fig. 3 shows the tracks of two displacement sensors, one in the top and the other in the bottom, fixed in the same vertical plane. Both of these tracks followed the same path showing a translatory movement of the bearing which is close to a harmonic motion. The frequency of the bearing motion was 2.96 Hz or 0.500282 of the shaft frequency.

## RESULTS AND DISCUSSION

A plain journal bearing with 50.963 mm diameter, 58 mm length, and 0.038 mm radial clearance was tested. The maximum out of round was 0.0012 mm which results in the ratio of the distortion amplitude to the radial clearance of 0.034. The test results confirm that the plain journal bearing is very susceptible to HFW. The whirl movement appeared as soon as the shaft started to turn (e.g. 355 RPM) and was an unstable motion because its orbit increased very rapidly and the shaft rubbed the bearing wall as can be seen in the bottom screen of Fig. 3.

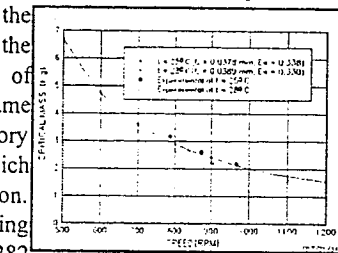


FIG. 4 PREDICTED AND MEASURED CRITICAL MASS OF A 3 WAVE BEARING

Then, a bearing circular profile was precisely altered such that a three wave bearing with a 0.0127 mm wave amplitude and a corresponding ratio of the wave amplitude to the main radial clearance of 0.343 was created. This bearing ran free of HFW up to a threshold speed between 740 and 960 RPM. Two sets of experiments were done; at 25 and 28°C respectively. The bearing mass was set at three different values (2.18, 2.6 and 3.17 kg.) The experimentally observed thresholds of HFW were plotted and compared to the theoretical prediction in Fig. 4. The plotted results of Fig. 4 show that the predicted results correspond to the observed results. Moreover, when the HFW phenomena starts, the orbit of the whirl is relatively small and the movement is quasi harmonic (e.g. in Fig. 5 at a speed of 921 RPM with a HFW frequency of 7.58 Hz, that corresponds to a frequency ratio of 0.498). Then, as speed increases, the whirl orbit also increases

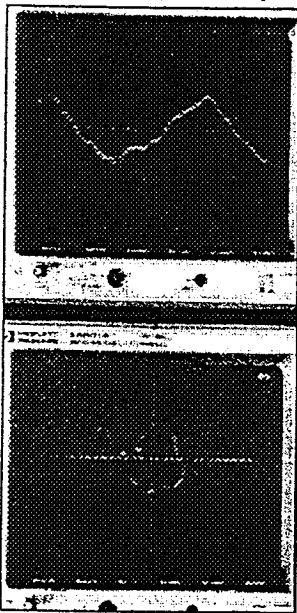


FIG. 5 HFW OF A 3 WAVE BEARING AT SPEED = 921 RPM

but in a stable manner, keeping a safe range inside the bearing clearance (see bottom screen in Fig. 6). In addition, as speed increases, the influence of the wave bearing profile is predominant by limiting the upper amplitude of the shaft center tracks (upper screen of Fig. 6) that modifies the shape of the orbit from circular to a three side orbit (bottom screen of Fig. 6).

## CONCLUDING REMARKS

1. The observed thresholds at which the HFW occurs correspond with the predicted thresholds.
2. An unloaded journal bearing with an altered circular profile, such as a wave bearing, allows operation over a range of speeds under which the bearing can run free of HFW. When this movement occurs the radius of the HFW increases up to a size where the equilibrium between the radial force generated by the whirl movement and the pressure force in the film is established. The equilibrium radius is smaller than the bearing clearance and the bearing can run stably and safely.

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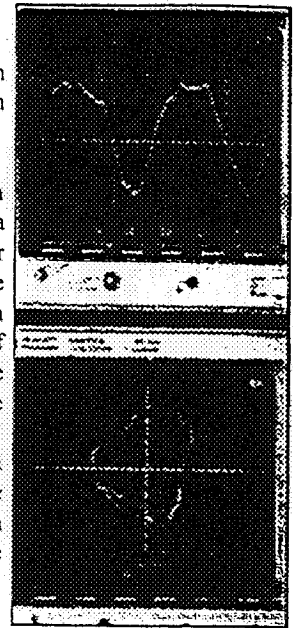


FIG. 6 HFW OF A 3 WAVE BEARING AT SPEED = 1391 RPM

# Three-Wave Gas Journal Bearing Behavior with Shaft Runout

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Robert C. Hendricks,

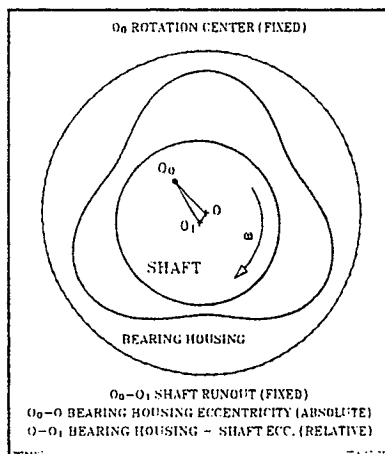
NASA Lewis Research Center, Cleveland, Ohio.

## ABSTRACT:

Experimental orbits of a free-mounted, three-wave gas journal bearing housing where recorded and compared to transient predicted orbits. The shaft was mounted eccentric with a fixed runout. Experimental observations for both the absolute bearing housing center orbits and the relative bearing housing center to shaft center orbits are in good agreement with the predictions. The sub-synchronous whirl motion generated by the fluid film was found experimentally and predicted theoretically for certain speeds. A three-wave journal bearing can run stably under dynamic loads with orbits well inside the bearing clearance. Moreover, the orbits are almost circular free of the influence of bearing wave shape.

## INTRODUCTION:

The wave bearing concept has been under development since 1992. At that time both the steady-state and dynamic performance under fixed side load were analyzed [1, 2]. Moreover, the influence of both the number of waves and the wave amplitude to radial clearance ratio were also analyzed [3, 4]. Since 1993, first the steady-state wave journal bearing characteristics and then the bearing dynamic stability have been experimentally measured; good agreement was found between the experimental data and theoretical predictions [5, 6, 7, and 8]. In addition the experimental work revealed good dynamic behavior of the wave bearing when the sub-synchronous whirl motion occurs. The wave bearing performed well keeping the orbit of the sub-synchronous motion inside the bearing clearance [7, 8]. Consequently, the wave bearing should perform well under dynamic loading conditions that often occur in most rotating machinery; any rotor can be subject to a dynamic load due to an unbalance or a runout of the shaft. Therefore in this paper both an experimental program and a transient analysis have been performed to record and respectively predict the orbits of the bearing housing center when the shaft has a known fixed runout.



## APPARATUS:

Fig. 1 Geometry of a Free-Mounted, Three-Wave Journal Bearing with Fixed Shaft Runout.

The wave bearing rig described in references [5 to 8] was used to perform the experimental work. The axis of the spindle that drives this rig was vertical and the experimental bearing housing was mounted on the rig table supported by two pressurized thrust plates. These configuration keeps the bearing housing stiff in the axial and angular directions but allows it to move freely in radial direction. The experimental shaft was an extension of the rig spindle shaft. It was mounted into the tapered end of the spindle shaft with a fixed runout (for this experiment  $11 \mu\text{m}$ ). A cross section by a horizontal plane of the experimental bearing is shown in Fig. 1. The fixed rotation center for the system is  $O_0$ .  $O_1$  and  $O$  are the centers of the shaft and bearing housing, respectively. The shaft runout  $O_0-O_1$  is fixed. The goal of this work is to record and to predict the absolute and relative orbits of the bearing housing center,  $O$ . The

motion of the center  $O$  can be observed like a absolute motion for instance with regard to the center of rotation  $O_o$ , or like a relative motion with regard to the center of the shaft  $O_1$ . Figure 2 shows the experimental bearing set up. Two sets of light beam proximity probes were used. Two probes were located at 90 degrees in the bottom side of the bearing housing and "looking" at the shaft. These probes detected the relative orbit of the bearing housing center, to the shaft center ( $O - O_1$ ). The second set of two probes were located also at 90 degrees but held by supports fixed on the rig table and "looking" at the bearing housing. These latter probes detect the absolute orbit of the bearing housing center ( $O - O_o$ ). A polished circumferential strip was made on the outside bearing housing surface to avoid asperity noise from its roughness. The light beam probes were calibrated using the known, fixed, runout of the shaft. The displacement of the shaft was measured with a precision of  $0.1 \mu\text{m}$ . The theoretical prediction of the orbits were made through a transient analysis of the bearing housing center motion.

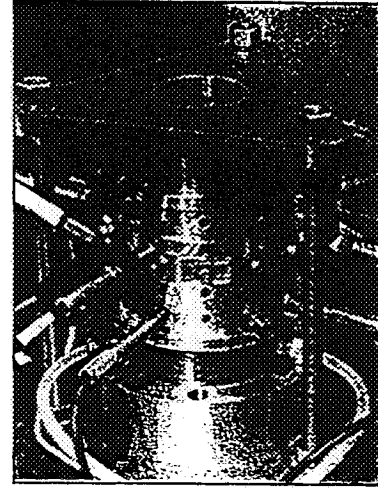


Fig. 2 Setup of a Free-Mounted, Three-Wave, Experimental Journal Bearing.

### ANALYSIS:

The study of the bearing housing center movement can be done using the motion equation of this center along and perpendicular to the radial direction  $O_o-O$  (axis  $\zeta$  and  $\eta$  in Fig.3):

$$\begin{aligned} MC \left[ \frac{d^2 \epsilon_o}{dt^2} - \epsilon_o \left( \frac{d\psi}{dt} \right)^2 \right] + K\epsilon_o + B \frac{d\epsilon_o}{dt} &= F_\zeta \\ MC \left( \epsilon_o \frac{d^2 \psi}{dt^2} + 2 \frac{d\psi}{dt} \frac{d\epsilon_o}{dt} \right) + K\epsilon_o \psi + B\epsilon_o \frac{d\psi}{dt} &= F_\eta \end{aligned} \quad (1)$$

where:  $M$  is the bearing housing mass,  $C$  is the radial clearance (difference between shaft radius and mean wave bearing),  $K$  is the external stiffness and  $B$  is the external damping in the bearing housing support and connection system,  $F_\zeta$  and  $F_\eta$  are the fluid film force components acting parallel and perpendicular to the  $O_o-O$  direction, respectively. Based on the notation defined in Fig. 3,  $\epsilon_o = e_o/C$  ( $e_o = O_o-O$ ) is the bearing housing absolute eccentricity ratio, and  $\psi$  is the rotation angle of the  $O_o-O$  around  $O_o$ . Also, Fig. 3 shows the eccentricity  $e = O_1-O$  ( $O_1-O$  is the line joining the shaft center,  $O_1$ , and the bearing center,  $O$ ), and the shaft runout  $\rho = O_o-O_1$ . Assuming that the motion starts from downward vertical where the shaft and the bearing are concentric ( $e = 0$ ), then, when the shaft rotates around  $O_o$  with the angular speed  $\omega$ ,  $\rho$  makes the angle  $\Omega$  and drives the bearing so that  $e_o$  makes the angle  $\psi$ .

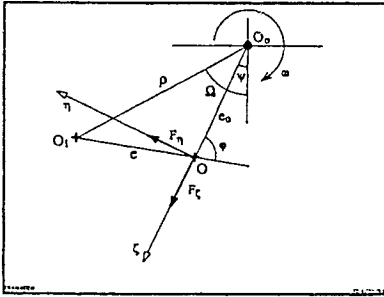


Fig. 3 Geometry of Rotation, Shaft, and Bearing Centers.

The governing equations (1) are two scalar, coupled, nonlinear ordinary differential equations. These equations are integrated simultaneously using a fourth order Runge-Kutta method for known values of  $M$ ,  $C$ ,  $F_r$ ,  $F_t$ ,  $K$ , and  $B$  and initial values of  $\epsilon_o$ ,  $\psi$ ,  $d\epsilon_o/dt$ , and  $d\psi/dt$  [9]. The fluid film forces applied to the bearing surface are:

$$\begin{aligned} F_\zeta &= F_r \cos \phi + F_t \sin \phi \\ F_\eta &= F_r \sin \phi - F_t \cos \phi \end{aligned} \quad (2)$$

$F_r$  and  $F_t$  are the projections of fluid film force along and perpendicular to the line of centers  $O_1-O$  (Fig. 3):

$$\begin{aligned}
F_r &= F_{r_0} + K_{rr}S_r + K_{rt}S_t + B_{rr}V_r + B_{rt}V_t \\
F_t &= F_{t_0} + K_{tr}S_r + K_{tt}S_t + B_{tr}V_r + B_{tt}V_t
\end{aligned}
\tag{3}$$

where  $S_i$ , and  $V_i$ , ( $i = r, t$ ) are the space (displacement) and velocity between the shaft center,  $O_1$ , and the bearing [housing] center,  $O$ , respectively. Coordinate  $r$  is along the line of centers,  $O_1-O$  (Fig. 3), and coordinate  $t$  is perpendicular to this line.  $F_{i0}$  ( $i = r, t$ ) are the bearing steady-state load projections, and  $K_{ij}$ ,  $B_{ij}$  ( $i = r, t$ ;  $j = r, t$ ) are the bearing dynamic stiffness and damping coefficients at a given time step location. The bearing steady-state force and dynamic stiffness and damping coefficients can be computed by integrating the Reynolds pressure equation at each time step location of the shaft with respect to the bearing. This equation, assuming the gas will expand isothermal, is:

$$\begin{aligned}
&\frac{\partial}{R\partial\theta}\left(\frac{h^3}{\mu}p\frac{\partial p}{R\partial\theta}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}p\frac{\partial p}{\partial z}\right) = \\
&6\left[2pV_n + 2\frac{\partial(ph)}{\partial t} + pV_\theta\frac{\partial h}{R\partial\theta} + h\frac{\partial(pV_\theta)}{R\partial\theta}\right]
\end{aligned}
\tag{4}$$

where:  $p$ , and  $h$  are the fluid film pressure and thickness, respectively;  $\theta$  is the angular coordinate along the shaft circumference, starting at the line of centers ( $O-O_1$ );  $z$  is the axial coordinate parallel to the shaft axis;  $R$  is the shaft radius;  $t$  is the time;  $V_n$  is the difference between the shaft surface and bearing surface speed projected on the perpendicular direction to the shaft or bearing surface, respectively, and  $V_\theta$  is the component of the shaft surface speed along its circumference.

The Reynolds equation (4) can be integrated using its complex form and a small perturbation technique. This procedure is described, for instance, in reference [10].

The solution procedure can start with input data set (bearing length, diameter, radial clearance, shaft turning speed, shaft runout, and the time step). In addition a set of starting values at time = 0 are required:

$$\begin{aligned}
\epsilon_o &= \rho, & \frac{\partial \epsilon_o}{\partial t} &= 0. \\
\psi &= 0., & \frac{\partial \psi}{\partial t} &= 0.
\end{aligned}
\tag{5}$$

Then, at each time step, knowing  $\epsilon_o$ ,  $\psi$ , and  $\Omega$  ( $\Omega = \omega t$ ), the  $O_1O_1O$  triangle (Fig. 3) is known and all geometrical parameters as well as displacements and velocities can be calculated. Therefore, the Reynolds equation (4) can be integrated over the fluid film. Then all parameters of the motion equation (1) are known as well as the starting values for the next time step ( $\epsilon_o$ ,  $\psi$ , and their time derivatives,  $\Omega$ ) and the procedure is repeated until the orbits are completed.

## RESULTS AND DISCUSSION:

The experimental bearing was 51 mm in diameter, 58 mm length, 20  $\mu\text{m}$  radial clearance and 2.2 kg mass. The bearing has three waves with a 0.5 wave amplitude to radial clearance ratio. The shaft was set with a 11  $\mu\text{m}$  runout. The external damping in the bearing housing support and connection system was found to be 0.05 Ns/m. The external stiffness ( $K$  in equations 1) have little influence on the bearings orbits and is approximately zero. The top

proximity probes (Fig. 2) produced 500 mV for 5.78  $\mu\text{m}$  and 4.78  $\mu\text{m}$  displacements in horizontal and vertical directions respectively, and the bottom probes produced 500 mV for 6.11  $\mu\text{m}$  and 6.90  $\mu\text{m}$  displacements in horizontal and vertical directions. (Horizontal and vertical directions refer to the directions on the oscilloscope photographs shown in the right side of the following 4 to 7 figures)

The test rig was run at four different speeds: 2156, 3288, 4588, and 5539 RPM. Up to 3100 RPM the bearing shows sensitivity to the sub-synchronous whirl. Both the absolute and relative observed orbits of the bearing housing center are shown in the oscilloscope photos shown on right side of figure 4. On the left side of figure 4 the computed orbits are presented with a time step of .00001 seconds and for 30000 steps. The experimental orbits appeared as ellipses rather than circles due to the difference in the probe sensitivity in horizontal and vertical directions mentioned above. First, Fig. 4 shows that both the experimental and theoretical orbits have the same patterns. These patterns are made by the sub-synchronous whirl motion. The transient analysis reveals these same patterns. Both the experimental and theoretical absolute orbits (top of Fig. 4) are within a region between 5  $\mu\text{m}$  to 12  $\mu\text{m}$  radius. Also, both the experimental and theoretical relative orbits (bottom of Fig. 4) run inside a circle of approx 5  $\mu\text{m}$  radius.

The bearing stability increased as the running speed of the rig increased. Figure 5 shows the results for 3288 RPM. The experimental orbits are perfectly stable. The shaft runout makes large absolute orbits of the bearing housing (right upper corner of Fig. 5). However, the radius of the relative orbits is approx. 2.5  $\mu\text{m}$  (bottom right corner of Fig. 5) despite the shaft 11  $\mu\text{m}$  runout, i.e., the bearing follows the shaft very well. The predicted orbits, showed by Fig. 5, match very well with the observed orbits. In addition, the theory shows that the bearing will run stably. After a couple of rotations from the starting point the orbits are stable keeping almost the same path.

The next runs were made at 4588 RPM and 5539 RPM (Fig. 6 and 7 respectively). The conclusions found running at 3288 RPM are also valid at higher speeds. However, the relative orbits of the bearing housing increase while the absolute orbits decrease as speed increases. This effect shows the influence of both the external damping and the bearing inertia on the orbit radius magnitude. In addition the bearing runs more and more stable as speed increases and the theory shows that the number of rotations before the bearing reaches a stable orbit diminishes as speed increases.

All runs (Fig. 4 to 7) show only a small influence of the bearing waved shape on the orbit shape despite the fact that the experimental bearing has a large wave amplitude ratio, 0.5. This confirms that the wave bearing with a low number of waves, such as 3, works well under dynamic loads. The bearing behaves in such a way as to average the influence of the wave even though locally the load is changing.

## CONCLUSIONS:

The experimental and theoretical work reported in this paper for a free mounted, three-wave gas journal bearing and a fix shaft runout, shows:

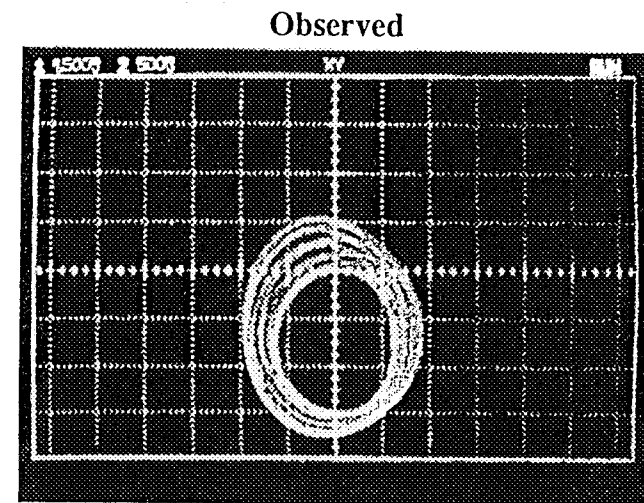
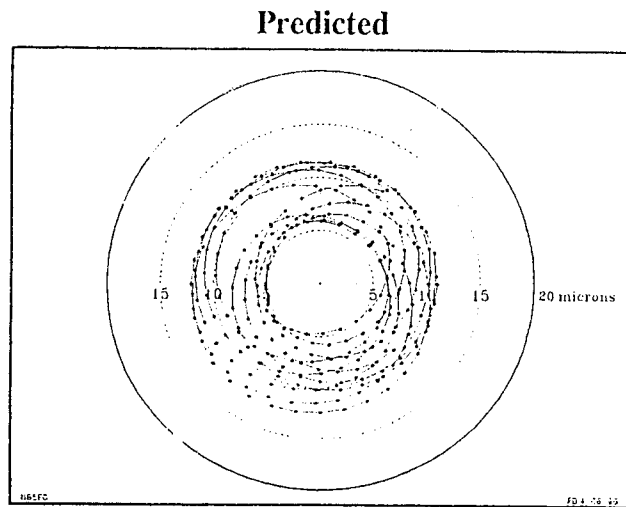
1. Good agreement between experimentally observed and theoretically predicted orbits at all tested speeds.
2. The sub-synchronous whirl motion influences the bearing housing orbits if the bearing speeds are in the region where the bearing itself is susceptible to the sub-synchronous whirl instability. When the bearing runs under such circumstances the orbits show a specific pattern. This was observed experimentally and was also confirmed theoretically by the transient analysis.
3. A three-wave journal bearing can run stably under dynamic rotating load, averaging its behavior when the wave exposure to the load is changing and with small orbits well within the bearing clearance. The orbits are almost circular and nearly free of the influence of wave bearing shape.

## ACKNOWLEDGMENT:

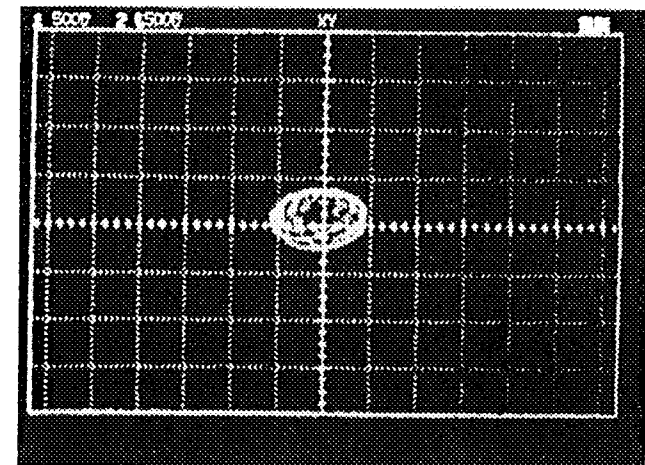
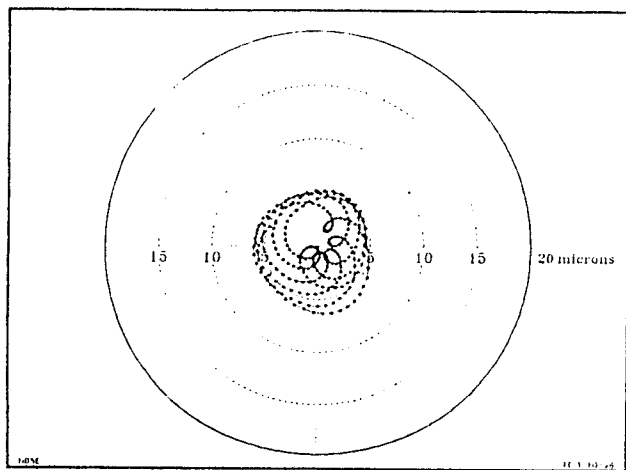
This work was performed under NASA NCC3-436 at NASA Lewis Research Center in Cleveland, OH. The authors would like to express their gratitude to Mr. Joseph Hemminger for continued interest and support.

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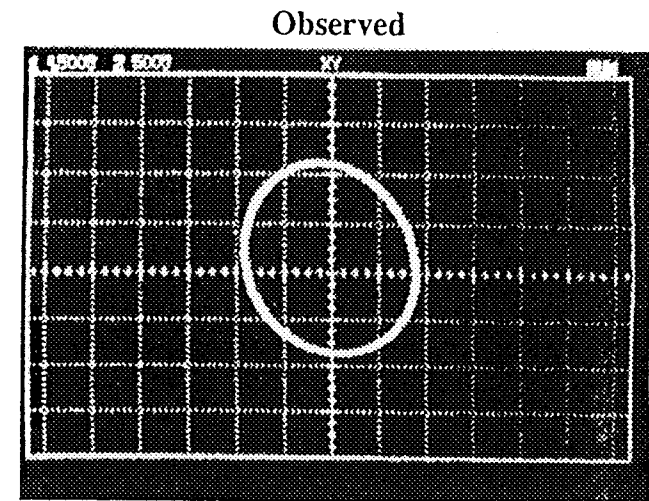
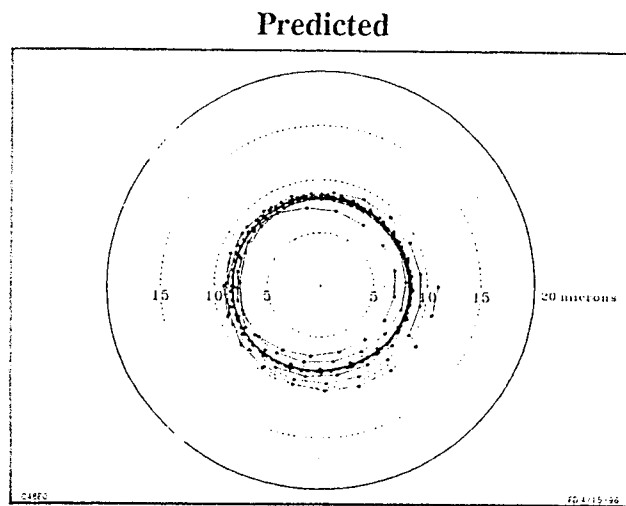
Absolute Bearing Housing Center Orbits



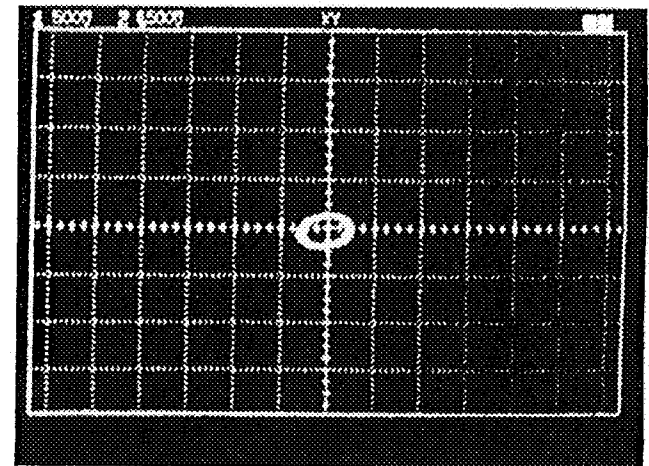
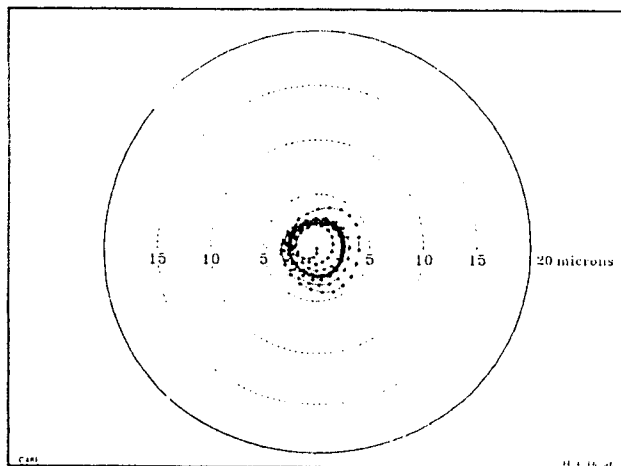
Relative Bearing Housing - Shaft Center Orbits

Fig. 4 Predicted and Experimentally Observed Orbits at 2156 RPM Shaft Rotating Speed





Absolute Bearing Housing Center Orbits

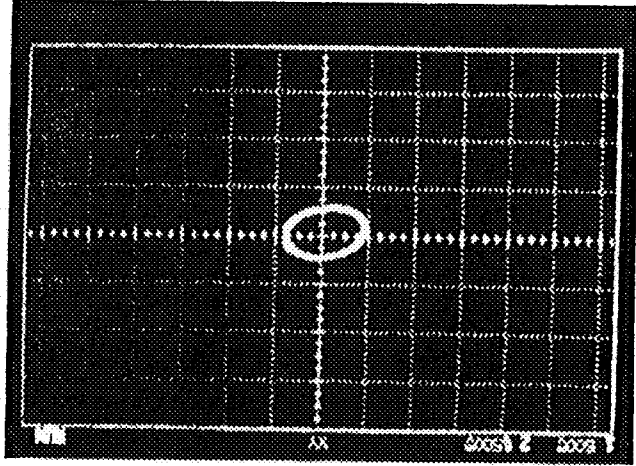
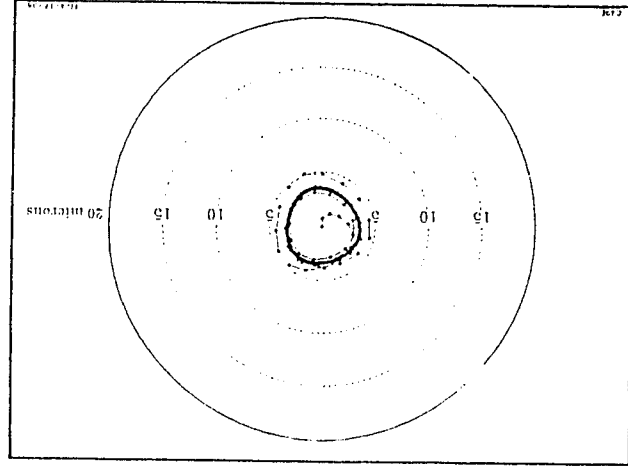


Relative Bearing Housing - Shaft Center Orbits

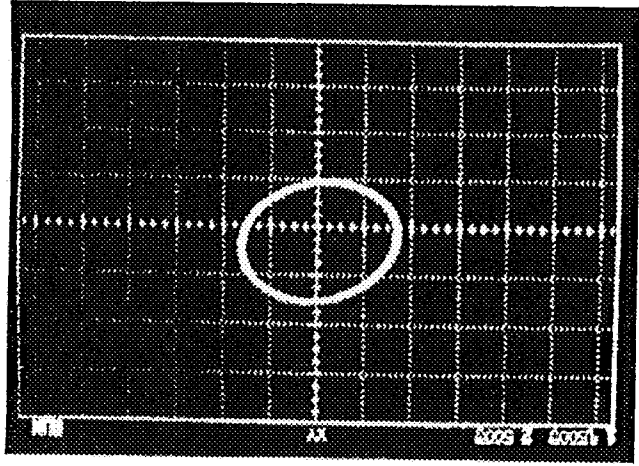
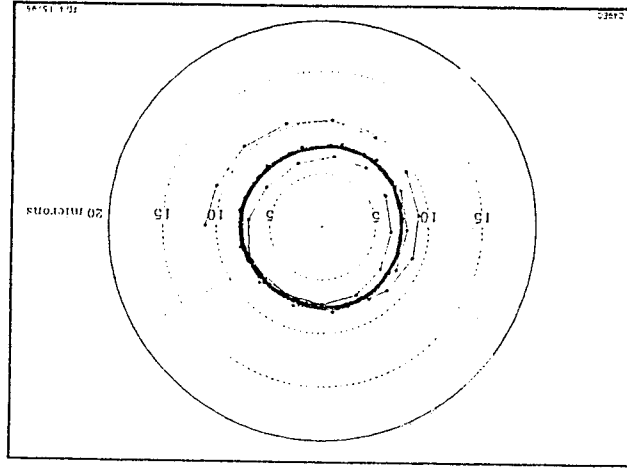
Fig. 5 Predicted and Experimentally Observed Orbits at 3288 RPM Shaft Rotating Speed

Fig. 6 Predicted and Experimentally Observed Orbits at 4588 RPM Shaft Rotating Speed

Relative Bearing Housing - Shaft Center Orbits

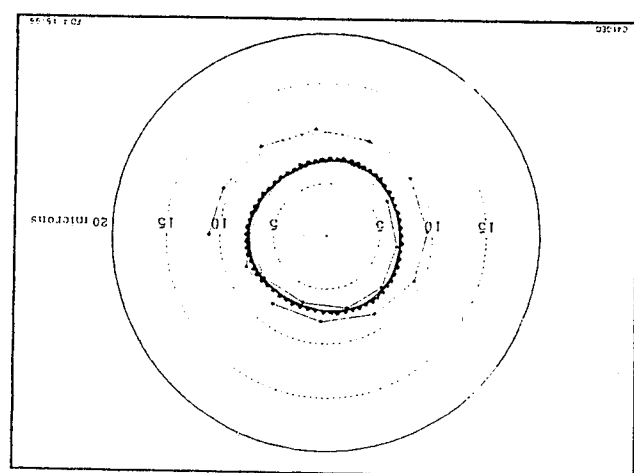
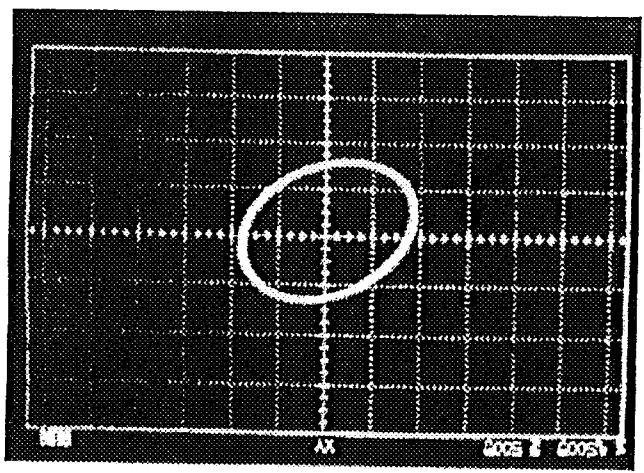
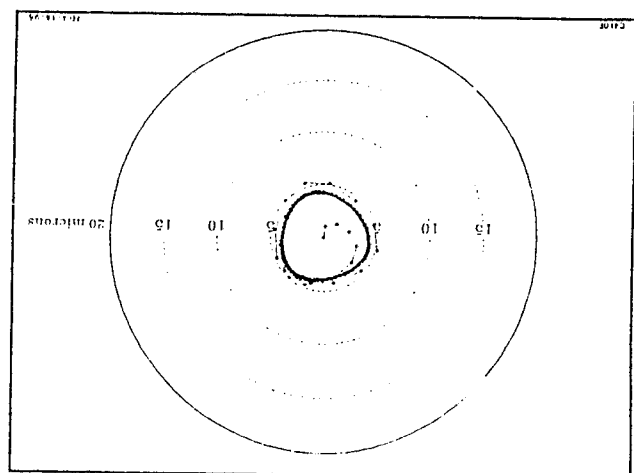
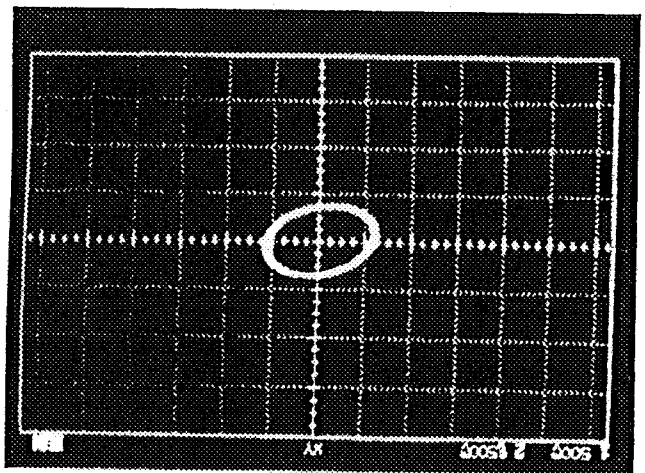


Absolute Bearing Housing Center Orbits



Observed

Predicted



Relative Bearing Housing - Shaft Center Orbits

Fig. 7 Predicted and Experimentally Observed Orbits at 5539 RPM Shaft Rotating Speed